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APPLICATION OF STATISTICAL TECHNIQUES TO MODEL SENSITIVITY TESTING

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Bethesda, Maryland

September 1974

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The advantages of factorial experimentation over the old classical One-Factor-At-A-Time experimentation for performing a sensitivity analysis on large scale computer models are discussed. An explanation is given of how time and effort can be saved in preparing inputs by choosing the proper sequence in which to make the model runs. Some ways in which the results can be analyzed and presented are also given.

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APPLICATION OF STATISTICAL TECHNIQUES TO MODEL SENSITIVITY TESTING

SEPTEMBER 1974

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APPLICATION OF STATISTICAL TECHNIQUES TO MODEL SENSITIVITY TESTING

SUMMARY

1. Facts. - An improved method of performing sensitivity analyses on large scale computer models was sought by the US Army Concepts Analysis Agency (CAA), 8120 Woodmont Avenue, Bethesda, MD 20014. The Methodology and Resources Directorate, CAA, initiated the task in FY 74, of developing, illustrating and documenting the needed improved method on a time available basis. The completion date of the task was September 1974.

2. Purpose. - The purpose of the task was to provide a comprehensive method of performing sensitivity analyses on large scale computer models. A method having greater efficiency than existing models was desired.

3. Objective. - A means of obtaining more information about how changes in input variables affect output variables in a large scale computer model was a prime objective. Minimization of the number of computer runs without loss of essential information was inherent in the task.

4. Discussion. - More and better information can be obtained by using factorial design of experiments, a statistical technique. A factorial design is a plan in which each factor under study is tested with each of the other factors at each level of interest. This method allows information to be obtained on the interaction effect of the factors under study. The interaction information is important because valid inferences cannot be made about any factor, independently of other factors if interaction exists. That is, each factor effect is dependent upon the level of the other factor(s). Frequently, because of the large number of factors and levels, a full (all factor level combinations) factorial design exceeds the number of computer runs available. The technique presented in this paper, for application of fractional factorial designs, reduces the number of computer runs required without sacrificing interaction information. The use and application of fractional factorial designs are discussed. An additional technique whereby time and effort in preparation of the inputs for the computer runs can be saved by choosing the run order is also discussed. While all techniques discussed above are widely used by statisticians and well documented in the statistical literature, the use of these techniques in model sensitivity testing is not widely used. Examples of output analyses and presentation of the results from a factorial designed experiment are presented to demonstrate how the method works.

5. <u>Observations</u>. - The application of fractional factorial designs provides a needed technique for reduction of computer runs without sacrificing model interaction information. Employment of an additional technique saves time and effort in preparation of inputs for computer runs. This latter technique, minimizes the factor level changes required in performing a factorial design experiment.

APPLICATION OF STATISTICAL TECHNIQUES TO MODEL SENSITIVITY TESTING

CHAPTER I
INTRODUCTION

1. Background. - The use of large scale computer models for guidance in real world decisions is now commonplace in the Department of Defense, other government agencies, and private industry. These models were developed from observations, data and theories to predict outcomes or results for conditions or situations that cannot be observed or in cases where costs of observation are excessive. These models can be broadly categorized as stochastic or deterministic. Stochastic or probablistic models attempt to capture the chance element associated with the processes being modeled, by using probability distributions for many of the input variables. Random samples are then drawn from the distributions and manipulated to produce model results. In deterministic models, the means or expected values of the variables are input as user-defined constants. These input variables are then used in a sequence of analytical equations or expressions to produce model results. In both stochastic and deterministic models, the question arises as to how sensitive the output factors are to changes in the input factors. Since these models are comprised of many factors with many interactions, it is very difficult, if not impossible, to analytically determine what effect a change in an input factor will have on an output factor. Therefore, the model is exercised with various changes to the input factors. This process is generally referred to as sensitivity analysis.

2. Purpose. - Design of experiment techniques have been widely used in the statistical community and is well documented in the literature. However, the use of these techniques for model sensitivity testing has not been common. A description of how more and better information can be obtained from sensitivity analysis by using factorial design of experiments is given in this paper. An explanation is given of how time and effort can be saved in preparing inputs by choosing the proper sequence in which to make the model runs. Some ways in which the results can be analyzed and presented are also given.

APPLICATION OF STATISTICAL TECHNIQUES TO MODEL SENSITIVITY TESTING

CHAPTER II
FACTORIAL DESIGN VERSUS ONE-FACTOR-AT-A-TIME

1. <u>One-Factor-At-A-Time</u>. - The old traditional method of doing a sensitivity analysis has been to set all the variables or factors except one at some nominal value and then vary one factor. A value higher or greater than the nominal value is input for one factor and the model is executed to determine the effect on the output variable(s). Then a value lower than the nominal value is input for the one factor and the model is rerun. Sometimes many different changes are made to the input variable and other times only two values, a high value and a low value, will be used. Frequently, many different variables will be tested, one at a time, each time with the remaining variables held fixed at a constant nominal value. Figure II-1 illustrates a One-Factor-At-A-Time sensitivity layout,

		Factor B			
	Level	1	2	3	4
Factor A	1		✓		
	2		✓		
	3		✓		
	4		✓		

FIGURE II-1, One-Factor-At-A-Time

where four levels (values) of factor A are tested at only one level (value) of factor B. While this One-Factor-At-A-Time method does provide some information on how the output varies for a change in the input factor or variable, the information is of a limited nature. First of all, all of the information obtained for factor A is for only one level of factor B. Information about the 12 vacant cells is not obtained. Thus, knowledge of how factor A would have affected the output if factor B would have been at some other level is not determined. Secondly, using the One-Factor-At-A-Time method, information concerning the effect of the interaction of factor A with factor B is not ascertainable. When there is an interaction effect between two factors the outcome is dependent upon which level each of the factors is set, i.e., one factor behaves differently under different levels of the other factor. In Figure II-2, for example, when factor B is set at the low level, the output variable decreases when factor A is changed from the low level to the high level. However, when factor B is set at the high level, the output variable increases when factor A is changed from the low level to the high level. (The lines in Figure II-2 are merely for connecting associated points.) Thus, factor A interacts with factor B.

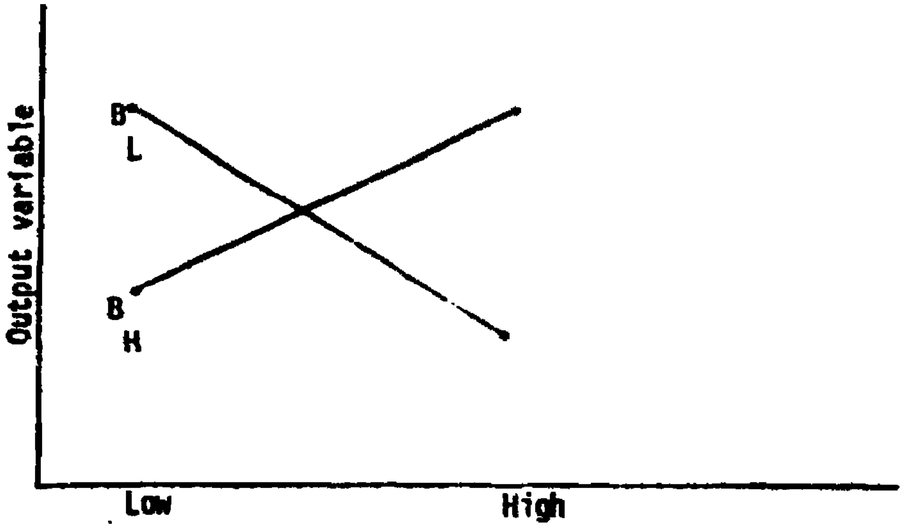


FIGURE II-2, Interaction Example

This interaction information is important since it tells the model user that neither factor A nor factor B can be examined in isolation due to the fact that the reaction of the output variable is dependent upon which level each of these factors was set. Therefore, it is desirable to employ a method which permits the assessment of at least the two-factor interactions.

2. Factorial Design Method

a. The proposed method is to use principles of experimental design which allow each variable (factor) under study to be examined with each of the other variables at each level of interest. In experimental design, this is referred to as a factorial design. For an illustrative example, consider three factors, A, B, and C, at two levels each, a low level (L) and a high level (H) (See Figure II-3). This example constitutes a 2^3 factorial experimental design.

Factor A		Factor B	Factor C: Low Level	Factor C: High Level
Low	Factor B	Low	LLL*	LLH
		High	LHL	LHH
High	Factor B	Low	HLL	HLH
		High	HHL	HHH

*L - Low Level H - High Level

FIGURE II-3, A 2^3 Factorial Experimental Design

All factors, other than A, B, and C, are held at a constant nominal value. The A, B, and C high/low values are chosen such that most of the values (90 or 95 percent) anticipated for that factor will be between these high/low values. By choosing high/low values near the extremes of the value spectrum, the probability of obtaining a good estimate of the trend in the effects will be increased. If the high/low values chosen are too close to each other, the probability of obtaining false trends or "noise" is increased.

b. Next "expand" the 2^3 factorial design into a 3^3 factorial design and call the third level the nominal level. This action will illustrate where the traditional method runs would fit in with those of the 2^3 factorial design (See Figure II-4). The quotation marks indicate the entries which belong to the traditional method. From this it can be seen how the traditional method runs are all clustered about the center of the design. The runs required for each method, in addition to the base case, are listed in Figure II-5 for ease of comparison. Note the factorial method requires two more runs than the traditional method. Next observe the output effects that can be assessed for each of the methods (See Figure II-6). Here it can be seen that for two additional runs, using the factorial method, information is obtained on the output effect of the AB, AC, BC, and ABC interactions in addition to the main factors (A, B, and C). In addition to being able to assess the interaction effects using the factorial method, more information is obtained on the factors at the points of most interest. Example: Observe factor A for some Measure of Effectiveness (MOE) for each method (See Figure II-7).

				Factor C		
				Low	Nominal	High
Factor A	Low	Factor B	Low	LLL*		LLH
			Nominal		"LNN"	
			High	LHL		LHH
	Nominal	Factor B	Low		"NLN"	
			Nominal	"NNL"	Base Case "NNN"	"NNH"
			High		"NHN"	
	High	Factor B	Low	HLL		HLH
			Nominal		"HNN"	
			High	HHL		HHH

*L - Low Level H - High Level N - Nominal Level

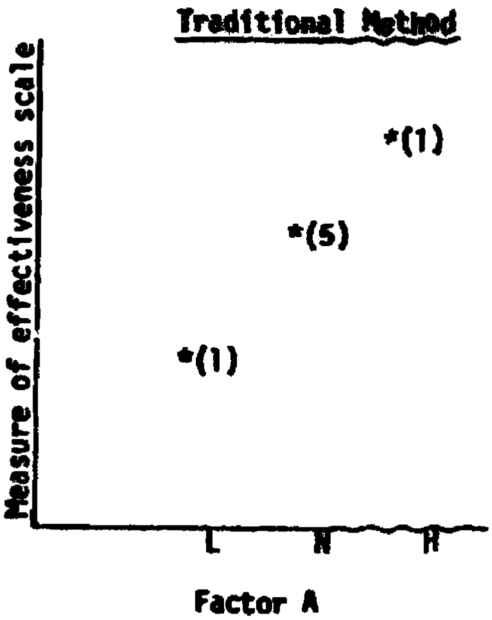
FIGURE II-4, "Expanded" Factorial Design

Run	Traditional method ABC	Run	Factorial method ABC
1	LNN	1	LLL
2	HNN	2	LLH
3	NLN	3	LHL
4	NHN	4	HLL
5	NNL	5	HHL
6	NNH	6	HLH
		7	LHH
		8	HHH

FIGURE II-5, Comparative Run Listing for Traditional Method and Factorial Method

Traditional method	Factorial method
A	A
B	B
C	C
	AB
	AC
	BC
	ABC

FIGURE II-6, Output Effects Assessable from Traditional Method and Factorial Method



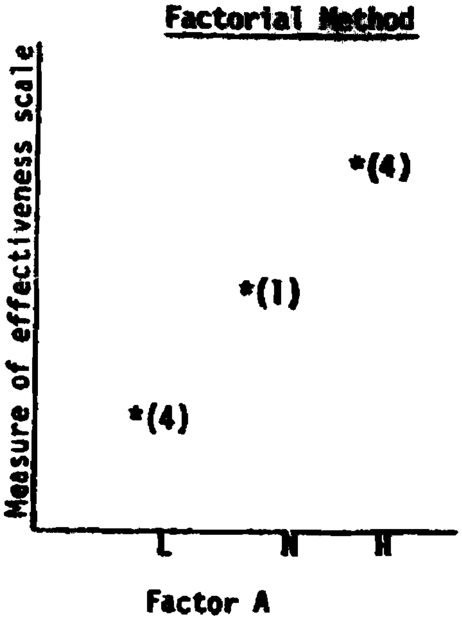


FIGURE II-7, Evaluation Points for Traditional and Factorial Methods

The numbers in parenthesis are the number of values used to estimate each point. For the traditional method, the results from only one run at the low value (LMM) and one run at the high value (HMM) are used for assessment. For the factorial method, the average of four values is used to assess the output effect at the low value (LLL, LLH, LHL and LHH). For the high value, the average of HLL, HHL, HLH and HHH, is used for assessment. Thus, it can be seen that four times as much data is used for the estimation of the values at the extremes (the high/low values) in the factorial method as in the traditional method. More data points are also used in estimating the high/low values for factors B and C. In addition to more data being used in the estimation of the high/low values, the interaction effects can be estimated for the AB, AC and BC interactions, through use of the factorial method.

CHAPTER III
FRACTIONAL FACTORIAL DESIGNS

1. Discussion. - As the number of factors to be considered in a factorial experiment increases, the number of factor level combinations increases very rapidly. For example, when there are five factors at two levels each, the number of combinations are $2^5 = 32$ computer runs for one replication. Recently, for A Tactical, Logistical, and Air Simulation (ATLAS) computer model (a theater-level ground combat simulation), it was desired to investigate 10 factors, which would have required $2^{10} = 1024$ computer runs for one replication. It is obvious that it would not be economically or timewise feasible to make 1024 computer runs for a sensitivity experiment. A method that is feasible is to use a design called fractional replication or fractional factorial. When only a fraction of a full design is run, the design is referred to as a fractional factorial design. The amount of fractionating that is performed depends on the number of factors under investigation, the number of levels of interest, the order of the interactions of interest and the total number of computer runs that it is feasible to perform. For this paper, consider only two levels, a low level and a high level. Therefore, only $1/2^p$ ($p = 1, 2, 3....$) fractional replicates, i.e., 1/2, 1/4 or 1/8, etc., of the full design, are employed.

2. Examples

a. For the ATLAS sensitivity experiment, in which there were 10 input factors of interest, it was determined that less than 150 runs could be made. After some research and study, it was found that a 1/8 fractional replicate of the 2^{10} experiment would take 128 runs and would provide the necessary information on the 10 main effects and the 45 two-factor interaction effects. By using a fractional factorial design, 7/8 of the runs (896) were eliminated

b. For another theater-level combat simulation model, the Conceptual Design for the Army in the Field Evaluation Model (CEM), available time and money limited the number of runs to 16. There were five primary factors of interest. A full factorial design would have required $2^5 = 32$ runs. Through use of a 1/2 fractional replicate, the number of runs was reduced to 16 and it was still possible to estimate the 5 main effects and the 10 two-factor interactions effects.

3. Caution. - It is emphasized that only a particular 1/2 of the runs in the 2^5 design or a particular 1/8 of the runs in the 2^{10} design will give the desired results. The proper runs are determined by choosing the defining contrasts such that none of the factors of interest are confounded with each other. These methods are explained in experimental design books, e.g., by Davies[1/], Kempthorne[2/] and Hicks[3/]. There is also a publication on Fractional Factorial Designs published by the National Bureau of Standards[4/] that gives various designs for factors at two levels.

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CHAPTER IV
MINIMIZING CHANGES TO FACTOR LEVELS

1. General Discussion

a. In designing a field experiment or a laboratory experiment, randomization of the order of experimentation is of utmost importance owing to the fact that certain variables cannot be controlled. Randomization of the order of experimentation will tend to average out the effect of the uncontrollable variables and eliminate any bias in the estimation of the effects of interest. In computer models, randomization of the order of runs is not necessary since the same answer will be obtained regardless of order (assuming the same random number seed is used). Since it is not necessary to randomize the order in which computer runs are made, it is advantageous to minimize the number of factor level changes and also to minimize the number of changes to the factor levels that are most difficult to change. Let's look at a 2^4 factorial design with the runs arranged in conventional Yates order (See Figure IV-1). In run number one, we see that all four factors are at the low level. Then in run number two, factor A has been changed to the high level, and the other three factors remain unchanged. In run number three, factor A has been changed back to the low level and factor B has been changed to the high level. Counting down each column the number of changes from Low (L) to High (H) or vice versa, it can be seen that 15 factor level changes occur in factor A, with 7, 3 and 1 changes for factors B, C and D, respectively, for a total of 26 factor level changes. The expected number of factor level changes for a 2^k factorial design in a completely randomized order is $(k)\ 2^{k-1}$. For a 2^4 design, the expected number of changes is equal to 32. The minimum number of factor level changes that is required for a 2^k factorial design is $2^k - 1$. For a 2^4 factorial design, the minimum number of changes required is 15. Thus, there is quite a difference in the expected number of changes and the minimum number of changes. There are over 200 different run orders that require only 15 factor level changes for a 2^4. Let's look at one of these (See Figure IV-2). Notice that for any two consecutive runs there is only a one factor level change. For example, factor A is changed from the low level in run number one to the high level in run number two with the other three factors remaining at the low level. In run number three, factor D is changed and the other three factors remain at the same level as in number two. Moving down each of the columns, note that six, two, four and three changes are required for factors A, B, C and D, respectively. If one factor requires more time or effort to change the parameters from low to high than the other factors, the columns can be rotated or interchanged without affecting the design or the total number of changes. For example, assume factor A is the most difficult to change from low to high. Currently, it is scheduled for the most changes. However, by interchanging the columns under A and B, factor A now has only two changes. This technique can be very useful when the factor under study is an entire array of numbers that is being changed by a certain amount.

b. The technique discussed, along with an algorithm for generating these designs for minimum changes is described in the February, 1974 issue of Technometrics.[3]

Run number	Factor A	Factor B	Factor C	Factor D
1	L	L	L	L
2	H	L	L	L
3	L	H	L	L
4	H	H	L	L
5	L	L	H	L
6	H	L	H	L
7	L	H	H	L
8	H	H	H	L
9	L	L	L	H
10	H	L	L	H
11	L	H	L	H
12	H	H	L	H
13	L	L	H	H
14	H	L	H	H
15	L	H	H	H
16	H	H	H	H
Number of factor level changes	15	7	3	1

FIGURE IV-1. A 2^4 Factorial Design with Runs Arranged in Conventional Yates Order.

Run number	Factor			
	A	B	C	D
1	L	L	L	L
2	H	L	L	L
3	H	L	L	H
4	H	L	H	H
5	H	H	H	H
6	L	H	H	H
7	L	H	L	H
8	H	H	L	H
9	H	H	L	L
10	L	H	L	L
11	L	H	H	L
12	H	H	H	L
13	H	L	H	L
14	L	L	H	L
15	L	L	H	H
16	L	L	L	H
Number of factor level changes	6	2	4	3

FIGURE IV-2, A 2^4 Factorial Design with 15 Factor Level Changes

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CHAPTER V
ANALYSES OF OUTPUT DATA

1. General. - If the model of interest is a probablistic model and the resources, such as time and money, permit, the experiment can be repeated to perform an analysis of variance (ANOVA) on the data. The hypothesis of no change in output values when the low level of a factor is changed to the high level of the factor can be statistically tested at a stated level of confidence. However, for a deterministic model, where repeating the experiment will give the same results, a within model variation does not exist. Therefore, the factors cannot be statistically tested. The following paragraphs describe some ways of analyzing the results from a deterministic model. Probablistic model results can also be analyzed in this manner, in addition to the ANOVA, or in place of the ANOVA if the experiment cannot be repeated.

2. Example

a. Main Effects. - For a look at the analysis, use the 2^3 design with some fictional data (See Figure V-1). To determine the effect of factor A on the MOE output, the average of the output data for factor A at the low level is obtained as follows:

$$\bar{A}_L = (LLL + LLH + LHL + LHH)/4$$
$$= (10 + 10 + 15 + 5)/4 = 10$$

Similarly for the high level of A:

$$\bar{A}_H = (HLL + HLH + HHL + HHH)/4$$
$$= (20 + 20 + 25 + 15)/4 = 20$$

The calculation of the B and C effects are done in a similar manner.

$$\bar{B}_L = (LLL + LLH + HLL + HLH)/4$$
$$= (10 + 10 + 20 + 20)/4 = 15$$

$$\bar{B}_H = (LHL + LHH + HHL + HHH)/4$$
$$= (15 + 5 + 25 + 15)/4 = 15$$

$$\bar{C}_L = (LLL + LHL + HLL + HHL)/4$$

$$= (10 + 15 + 20 + 25)/4 = 17.5$$

$$\bar{C}_H = (LLH + LHH + HLH + HHH)/4$$

$$= (10 + 5 + 20 + 15)/4 = 12.5$$

Figure V-2 is a graphical illustration of the A, B and C effects.

			Factor C: Low	Factor C: High
Factor A: Low	Factor B	Low	LLL* 10	LLH 10
	Factor B	High	LHL 15	LHH 5
Factor A: High	Factor B	Low	HLL 20	HLH 20
	Factor B	High	HHL 25	HHH 15

*L - Low level H - High level

FIGURE V-1, A 2^3 Factorial Design with Fictional Data

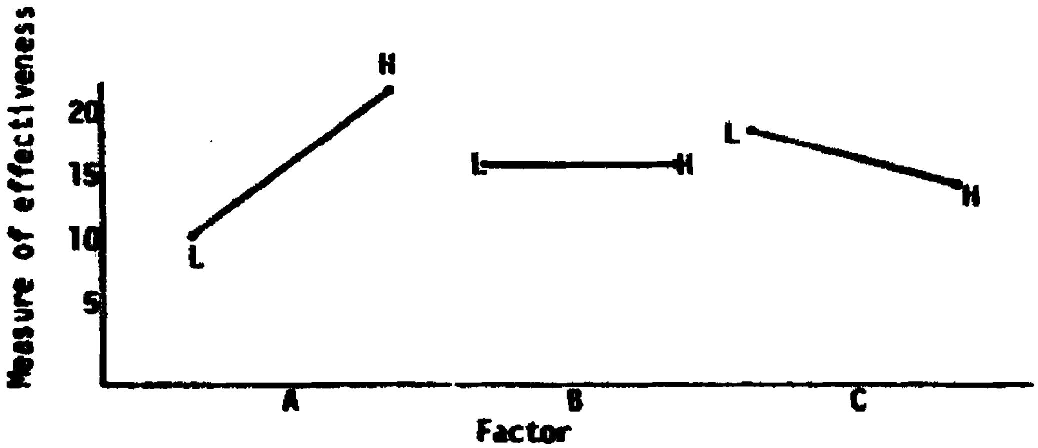

FIGURE V-2, Graphical Illustration of A, B and C Effects

Thus, changing from the low level to the high level caused a sharp increase in the MOE value attributed to A, no change attributed to B and a moderate decrease attributed to factor C. (The lines in Figure V-2 are merely for connecting associated points.) By subtracting the low level from the high level a numerical measure can be obtained for comparison. For the three factors this measure would be:

$$\bar{A}_H - \bar{A}_L = 20 - 10 = 10$$

$$\bar{B}_H - \bar{B}_L = 15 - 15 = 0$$

$$\bar{C}_H - \bar{C}_L = 12.5 - 17.5 = -5.0$$

b. Interaction Effect. - The AB interaction term mean is calculated as follows:

$$\overline{A_L B_L} = (LLL + LLH)/2 = (10 + 10)/2 = 10$$

$$\overline{A_L B_H} = (LHL + LHH)/2 = (15 + 5)/2 = 10$$

$$\overline{A_H B_L} = (HLL + HLH)/2 = (20 + 20)/2 = 20$$

$$\overline{A_H B_H} = (HHL + HHH)/2 = (25 + 15)/2 = 20$$

The change in factor B when factor A is at the low level is

$(\overline{A_L B_L} - \overline{A_L B_H}) = (10-10) = 0$, and the change in factor B when factor A is at the high level is $(\overline{A_H B_L} - \overline{A_H B_H}) = (20-20) = 0$. The AC interaction term mean is calculated as follows:

$$\overline{A_L C_L} = (LLL + LHL)/2 = (10 + 15)/2 = 12.5$$

$$\overline{A_L C_H} = (LLH + LHH)/2 = (10 + 5)/2 = 7.5$$

$$\overline{A_H C_L} = (HLL + HHL)/2 = (20 + 25)/2 = 22.5$$

$$\overline{A_H C_H} = (HLH + HHH)/2 = (20 + 15)/2 = 17.5$$

The change in factor C when factor A is at the low level is, $(\overline{AC}_{LL} - \overline{AC}_{LH}) = (12.5 - 7.5) = 5.0$, and the change in factor C when factor A is at the high level is, $(\overline{AC}_{HL} - \overline{AC}_{HH}) = (22.5 - 17.5) = 5.0$. The BC interaction term mean is calculated as follows:

$$\overline{BC}_{LL} = (LLL + HLL)/2 = (10 + 20)/2 = 15$$

$$\overline{BC}_{LH} = (LLH + HLH)/2 = (10 + 20)/2 = 15$$

$$\overline{BC}_{HL} = (LHL + HHL)/2 = (15 + 25)/2 = 20$$

$$\overline{BC}_{HH} = (LHH + HHH)/2 = (5 + 15)/2 = 10$$

The change in factor C when factor B is at the low level is,

$(\overline{BC}_{LL} - \overline{BC}_{LH}) = (15 - 15) = 0$, while the change in factor C when factor B is at the high level is $(\overline{BC}_{HL} - \overline{BC}_{HH}) = (20 - 10) = 10$. The three interaction term effects can be graphically illustrated as in Figure V-3. (The lines in Figure V-3 are merely for connecting associated points.) Since the lines are parallel, it can be seen that there is absolutely no interaction between factors A and B or factors A and C. When factor B is at the low level, there is no change in the Measure of Effectiveness value as factor C is changed from the low level to the high level. However, when factor B is at the high level, there is a decrease in the Measure of Effectiveness value as factor C is changed from the low level to the high level. Thus, one factor (factor C) behaves differently under different levels of the other factor (factor B). As there is an interaction effect between factors B and C, any further testing involving either factor B or factor C will have to include the other factor since factors B and C are not independent of each other. It should be noted that the results are seldom, if ever, as clear cut as the demonstration example. The lines are seldom exactly parallel as was demonstrated. However, the degree of non-parallelism can be noted and rank ordered in those cases that are not clear cut.

c. <u>Result</u>. - From the example given, it can be concluded that the Measure of Effectiveness value is most sensitive to factor A, with factor C being the next most sensitive, and factor B being important due to the fact that it interacts with factor C.

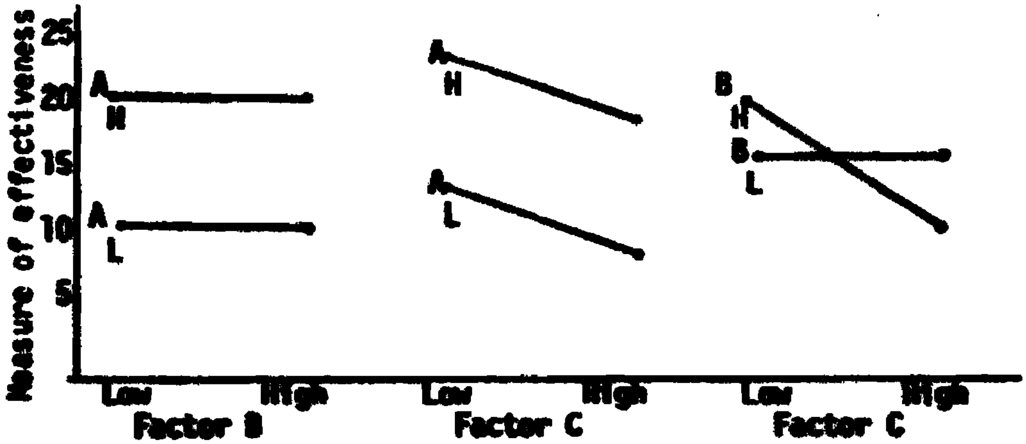

FIGURE V-3. Graphical Illustration of AB, AC and BC Interaction Effects

CHAPTER VI
CONCLUSION

1. Advantages of Experimental Design. - It is hoped that the demonstration shows how a statistical tool, namely the design of experiments, can be used to obtain more and better information from a sensitivity analysis of a model than the old traditional One-Factor-At-A-Time approach. In very small models where it can be assumed that all the factors are independent and that interactions will not occur, One-Factor-At-A-Time sensitivity testing would be acceptable. However, in the large models which were built to represent complex processes involving many interrelated parts or components which may have an interacting effect upon output variables, One-Factor-At-A-Time analysis is not the proper tool. It is believed that use of the design of experiments techniques leads to a better understanding of model sensitivity to specific factors and combinations of factors.

APPLICATION OF STATISTICAL TECHNIQUES TO MODEL SENSITIVITY TESTING

APPENDIX A
STUDY TEAM

1. <u>Study Director</u>

   Jerry Thomas, Methodology and Resources Directorate

2. <u>Support Personnel</u>

   Geraldine B. Sabo

   Carol S. Martin

APPENDIX B
CITED REFERENCES

1. Davies, O. L., Design and Analysis of Industrial Experiments, Hafner Publishing Company, New York, 1954.

2. Kempthorne, O., The Design and Analysis of Experiments, John Wiley & Sons, Inc., New York, 1952.

3. Hicks, Charles R., Fundamental Concepts in the Design of Experiments, Holt, Rinehart and Winston, New York, 1964.

4. Fractional Factorial Experiment Designs for Factors at Two Levels, NBS Applied Mathematics Series 48, US Government Printing Office, Washington, DC, Apr 57.

5. Dickinson, A. W., "Some Run Orders Requiring a Minimum Number of Factor Level Changes for the 2^4 and 2^5 Main Effect Plans," Technometrics, Volume 16, Number 1, Feb 74, pp 31-37.